PHYSICS 232
2013/14 Term 2

Final Examination

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Free Response: Write out complete answers to the following questions. Show your work.

(10$^{\text{pts}}$) **1.** Consider the periodic function $f(x) = |\sin x|$ which, on the interval pi $-\pi$ $< x < \pi$ can be expressed as:

$$f(x) = \begin{cases} -\sin x, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$$

(a) Sketch several periods of $f(x)$. Be sure to include scales for both the $x-$ and $y-$axes of your plot. (**2 marks**)

(b) Find the Fourier series for this function. Simplify your answers as much as possible. Write out the first five non-zero terms of the Fourier series. (**8 marks)**)

You may make use the following relations:

$$\int_{-\pi}^{0} \sin x \cos nx \, dx = -\int_{0}^{\pi} \sin x \cos nx = \frac{\cos n\pi + 1}{\cancel{(n+1)(n-1)}} \quad n^2-1$$

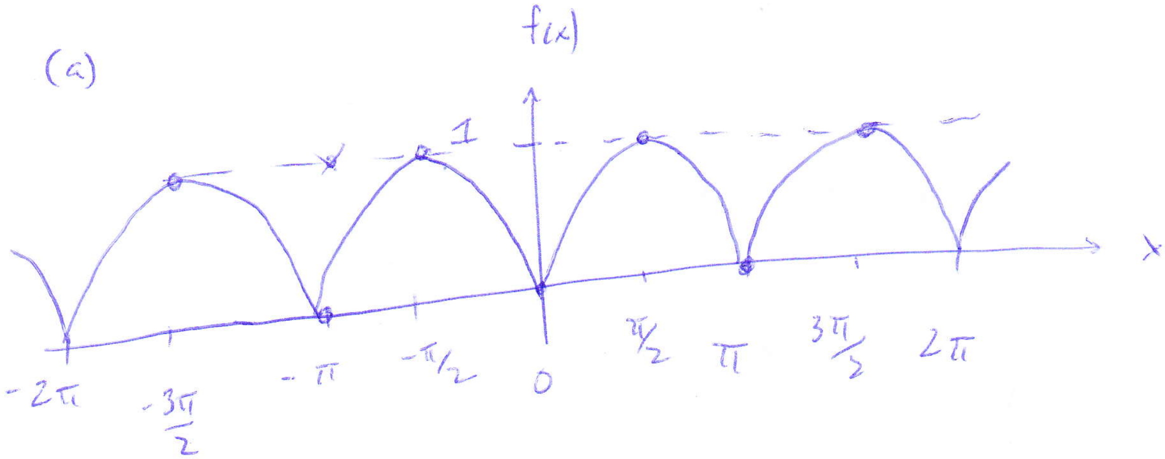
$$\int_{-\pi}^{0} \sin x \sin nx \, dx = \int_{0}^{\pi} \sin x \sin nx \, dx = -\frac{\sin n\pi}{n^2 - 1}$$

for $n \neq 1$ and:

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

(a)



10 pts

Sol'ns

(b) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$= \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \dots$

$+ b_1 \sin x + b_2 \sin 2x + \dots$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$

start w/ a_0

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\sin x \, dx + \int_{0}^{\pi} \sin x \, dx\right]$

$= \frac{1}{\pi}\left[\cos x \Big|_{-\pi}^{0} - \cos x \Big|_{0}^{\pi}\right]$

$= \frac{1}{\pi}\left[1-(-1) - (-1-1)\right] = \frac{4}{\pi}$

$a_1 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\sin x \cos x \, dx + \int_{0}^{\pi} \sin x \cos x \, dx\right]$

$= \frac{1}{\pi}\left[-\frac{1}{2}\int_{-\pi}^{0} \sin 2x \, dx + \frac{1}{2}\int_{0}^{\pi} \sin 2x \, dx\right]$

$= \frac{1}{2\pi}\left[+\frac{\cos 2x}{2}\Big|_{-\pi}^{0} - \frac{\cos 2x}{2}\Big|_{0}^{\pi}\right] = \frac{1}{4\pi}\left[1-1-(1-1)\right] = 0$

0 pts

$n > 1$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\sin x \cos nx \, dx + \int_{0}^{\pi} \sin x \cos nx \, dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{\cos n\pi + 1}{(n+1)(n-1)} - \frac{\cos n\pi + 1}{(n+1)(n-1)}\right] = -\frac{2}{\pi}\frac{\cos n\pi + 1}{(n+1)(n-1)}$$

n	a_n
0	$4/\pi$
1	0
2	$-\frac{4}{3\pi}$
3	0
4	$-\frac{4}{15\pi}$
5	0
6	$-\frac{4}{35\pi}$
7	0
8	$-\frac{4}{63\pi}$

check b_1

$$b_1 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\sin^2 x \, dx + \int_{0}^{\pi} \sin^2 x \, dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{1}{2}\int_{-\pi}^{0} (1-\cos 2x)\, dx + \frac{1}{2}\int_{0}^{\pi} (1-\cos 2x)\, dx\right]$$

$$= \frac{1}{2\pi}\left[-\left(x - \frac{\sin 2x}{2}\right)\Big|_{-\pi}^{0} + \left(x - \frac{\sin 2x}{2}\right)\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[-(0-(-\pi)) + (\pi - 0)\right] = 0$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\sin x \sin nx \, dx + \int_{0}^{\pi} \sin x \sin nx \, dx\right]$$

$$= \frac{1}{\pi}\left[+\frac{\sin n\pi}{n^2-1} - \frac{\sin n\pi}{n^2-1}\right] = 0$$

$\therefore b_n = 0 \quad \forall n$

$$\boxed{\therefore f(x) = \frac{2}{\pi} - \frac{4}{3\pi}\cos 2x - \frac{4}{15\pi}\cos 4x - \frac{4}{35\pi}\cos 6x - \frac{4}{36\pi}\cos 8\pi - \dots}$$

(10[pts]) **2.** Because statistically there is a certain number of people that will fail to show up for flights, airlines routinely overbook their flights. Assume that there is a 3% chance that any given passenger booked on a flight will be a no-show.

(a) For a particular flight from Vancouver to Winnipeg the plane has a 100 person capacity. Indicate which of the Gaussian, binomial, or Poisson distributions accurately describes the probability distribution of no-shows for the flight? (List all of the valid choices, there may be more than one). (**2 marks**)

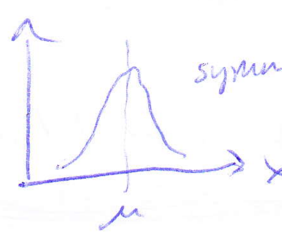
(b) If the airline sells 3 extra tickets for the flight, what is the probability that the flight will be overbooked? (**5 marks**)

(c) If this flight is offered 10 times, what is the probability that it is overbooked at least 3 times? (**3 marks**)

(a) Binomial dist'n will work. Poisson & Gaussian dist'ns are special cases of binomial.

Poisson valid when $p \ll 1$. This problem has prob. on no show $p = 0.03 \ll 1$ ∴ poisson valid.

Gaussian dist'n requires $\mu = np \gg 1$. Here $np \approx 100(0.03) = 3$ which doesn't satisfy $np \gg 1$ ∴ cannot use Gaussian dist'n.



symmetric, so μ must be far from x=0 position.

(b) Here $n = 103$ (no. of tickets sold)

start w/ Poisson dist'n $P_P = \frac{\mu^x e^{-\mu}}{x!}$ μ is mean no. of no shows $\mu = (103)(0.03) = 3.09$

x is no. of successes (i.e. no. of no shows).

Overbooked if $x = 0, 1, 2$

$P_0 = \frac{\mu^0 e^{-\mu}}{0!} = e^{-\mu}$

$P_1 = \mu e^{-\mu}$

$P_2 = \frac{\mu^2 e^{-\mu}}{2}$

~~$P_3 = \frac{\mu^3 e^{-\mu}}{6}$~~

∴ $P_{overbooked} = P_0 + P_1 + P_2 = e^{-\mu}\left(1 + \mu + \frac{\mu^2}{2}\right)$

$= \cancel{0.62}$ → ~~62% chance~~

$\approx .403$ 40% chance

10 pts

Alternatively can use binomial dist'n

$$P_B = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$P_0 = \frac{103!}{103!} p^0 (1-p)^{103} = (0.97)^{103} = 0.043$$

$$P_1 = \frac{103!}{102!} p^1 (1-p)^{102} = 103p(1-p)^{102} = 0.138$$

$$P_2 = \frac{103 \cdot 102}{2} p^2 (1-p)^{101} = \cancel{0.436}\ .218$$

$$\therefore P_{overbooked} = P_0 + P_1 + P_2 = \cancel{61.7}\ .399 \rightarrow \cancel{62\%}\ 40\%$$

(c) For this prob. prob that a particular flight is over booked is $p = \cancel{0.62}$ 0.399 which is not much less than 1. $\therefore$ have to use binomial dist'n.

$$P_B = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

no. success k is no. of times ~~overbooked~~ overbooked

$\cancel{p = 0.62}$ $\cancel{p = (1-0.62) = .38}$ $p = \cancel{0.62}\ .399$

overbooked zero times $P_0 = \frac{10!}{0!10!} p^0 (1-p)^{10} = (1-p)^{10} = \cancel{6.28\times10^{-5}}\ 6.15\times10^{-3}$

" one time $P_1 = 10p(1-p)^9 = \cancel{9.22\times10^{-4}}\ \cancel{1.02\times10^{-3}}\ 4.08\times10^{-2}$

" two times $P_2 = \cancel{9\cdot 8}\ \frac{10\cdot 9}{2} p^2 (1-p)^8 = \cancel{1.50\times10^{-2}}\ \cancel{7.52\times10^{-3}}\ 0.122$

$\therefore$ Prob. that overbooked 3 or more times is

$$1 - P_0 - P_1 - P_2 = \cancel{0.997}\ \cancel{0.984}\ 0.831$$

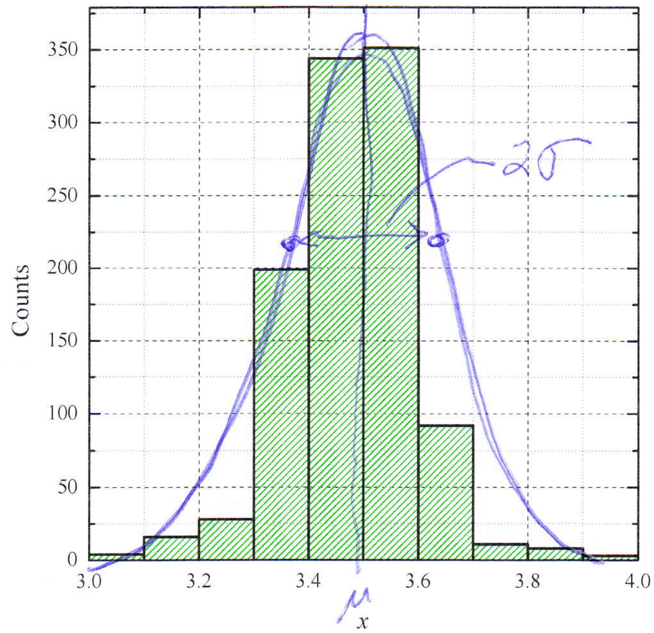
v. high.

Sol'n 1

(10pts) **3.** The Gaussian distribution is given by:

$$P_G(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

The figure below shows some data that are approximately Gaussian distributed.



(a) What is the numerical value of $P_G(x=\mu\pm\sigma)/P_G(x=\mu)$? Show your work. (**2 marks**)

(b) Estimate the value of μ and σ of the distribution shown in the figure above. Draw the approximate shape of the Gaussian distribution that the data in the histogram follow. (**4 marks**)

(c) The distribution in the figure is made up of $N = 1061$ individual measurements. What is the approximate error in the mean value estimated from the sample distribution? (**4 marks**)

(a) $P_G(x=\mu\pm\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\mu\pm\sigma-\mu)^2}{2\sigma^2}\right] = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\right)$

$P_G(x=\mu) = \frac{1}{\sqrt{2\pi}\sigma}\exp[0] = \frac{1}{\sqrt{2\pi}\sigma}$

$\therefore \frac{P_G(x=\mu\pm\sigma)}{P_G(x=\mu)} = e^{-1/2} = \boxed{0.607}$

10 pts

(b) peak of dist'n approximately at

$x = 3.5 \quad \therefore \boxed{\mu \approx 3.5}$

Peak height $\approx 360 \quad \therefore (0.607)(360) = 218$

$\therefore 2\sigma \approx 3.65 - 3.35 = 0.3$

$\boxed{\therefore \sigma \approx 0.15}$

(c) $\sigma_\mu = \dfrac{\sigma}{\sqrt{N}} = \dfrac{0.15}{\sqrt{1061}} = 4.6 \times 10^{-3}$

$\therefore \mu = 3.5 \pm 0.005$

0 pts

soln's

(10pts) **4.** You have three resistors with specified resistances and uncertainties: $R_1 \pm \sigma_1$, $R_2 \pm \sigma_2$, and $R_3 \pm \sigma_3$.

(a) If the three resistors are connected in series, the equivalent resistance is given by:

$$R_s = R_1 + R_2 + R_3$$

Find an expression for the uncertainty in R_s (σ_s) in terms of R_1, R_2, R_3 and their uncertainties. (**2 marks**)

(b) If the three resistors are connected in parallel, the equivalent resistance is given by:

$$R_p = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

Find an expression for the uncertainty in R_p (σ_p) in terms of R_1, R_2, R_3 and their uncertainties. (**5 marks**)

(c) Suppose you want to make a 300 Ω resistor. Given the limited equipment that you have in the lab, your options are to combine three 100 Ω ± 5% resistors in series or to combine three 900 Ω ± 5% resistors in parallel. Compare the resulting numerical values of σ_s and σ_p. (**3 marks**)

(a)

$$\sigma_s^2 = \sum \left(\frac{\partial R_s}{\partial R_i}\sigma_i\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$$

$$\therefore \sigma_s = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$$

(b)

$$\sigma_p^2 = \sum \left(\frac{\partial R_p}{\partial R_i}\sigma_i\right)^2$$

$$\frac{\partial R_p}{\partial R_1} = -\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-2} \frac{\partial}{\partial R_1}\left(\frac{1}{R_1}\right)$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-2} \frac{1}{R_1^2}$$

$$= \frac{1/R_1^2}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^2}$$

likewise for R_2 & R_3 derivatives

$$\therefore \sigma_p^2 = \frac{\left(\frac{\sigma_1}{R_1^2}\right)^2 + \left(\frac{\sigma_2}{R_2^2}\right)^2 + \left(\frac{\sigma_3}{R_3^2}\right)^2}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^4}$$

10 pts

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$$\therefore \sigma_p = \frac{\sqrt{\left(\frac{\sigma_1}{R_1^2}\right)^2 + \left(\frac{\sigma_2}{R_2^2}\right)^2 + \left(\frac{\sigma_3}{R_3^2}\right)^2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^2}$$

(c) series: $\sigma_1 = \sigma_2 = \sigma_3 = 0.05(100\,\Omega) = 5\,\Omega$

$$\therefore \sigma_s = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} = \boxed{8.66\,\Omega}$$

parallel: $\sigma_1 = \sigma_2 = \sigma_3 = 0.05(900\,\Omega) = 45\,\Omega$

$$\left(\frac{\sigma_1}{R_1^2}\right)^2 = 3.086\times10^{-9}\,\Omega^{-2} = \left(\frac{\sigma_2}{R_2^2}\right)^2 = \left(\frac{\sigma_3}{R_3^2}\right)^2$$

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 3.333\times10^{-3}\,\Omega^{-1}$$

$$\therefore \sigma_p = 8.66\,\Omega$$

$$\therefore \boxed{\sigma_s = \sigma_p}$$

0 pts

(10pts) **5.** In an experiment to measure the work function W of tungsten, one measures the electron current I as a function of the tungsten temperature T. Theoretically, these variables are expected to be related by the equation:

$$\frac{I}{A} = BT^2 \exp\left(\frac{-W}{k_B T}\right)$$

where A is the surface are of the tungsten sample, k_B is Boltzmann's constant, and B is a constant. Assume that Boltzmann's constant is known to within some uncertainty σ_{k_B} (i.e., $k_B \pm \sigma_{k_B}$ is known). However, B is an unknown constant and the tungsten sample is an odd shape so that its surface area A is also unknown.

(a) Linearize the equation above such that the work function W can be extracted from the slope of a straight line. Give the equation of the straight line and describe the plot (y vs x) that you would generate. What does y represent and what does x represent? (**7 marks**)

(b) If slope of your graph and its uncertainty ($m \pm \sigma_m$) are determined from a linear fit, how would you determine the uncertainty in the work function σ_W? (**3 marks**)

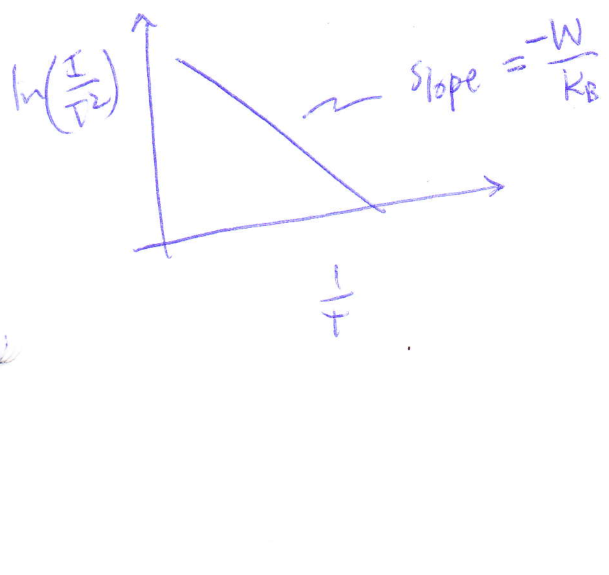
(a) $\frac{I}{A} = BT^2 \exp\left(-\frac{W}{k_B T}\right)$ $\therefore$ $I = ABT^2 \exp\left(-\frac{W}{k_B T}\right)$

$\therefore$ $\frac{I}{T^2} = AB \exp\left(-\frac{W}{k_B T}\right)$ $y \to \ln\left(\frac{I}{T^2}\right)$

$\therefore$ $\ln\left(\frac{I}{T^2}\right) = \ln(AB) - \frac{W}{k_B T}$ $x \to \frac{1}{T}$

Plot $\ln\left(\frac{I}{T^2}\right)$ vs $\frac{1}{T}$ $\Rightarrow$ slope is $-\frac{W}{k_B}$

y-intercept is $\ln(AB)$



10 pts

 Name: Jake Bobowski

Sol'ns

$$m = -\frac{W}{k_B}$$

$$\therefore \boxed{W = -k_B m}$$

$$\therefore \sigma_W = \sqrt{\left(\frac{\partial W}{\partial k_B}\sigma_{k_B}\right)^2 + \left(\frac{\partial W}{\partial m}\sigma_m\right)^2}$$

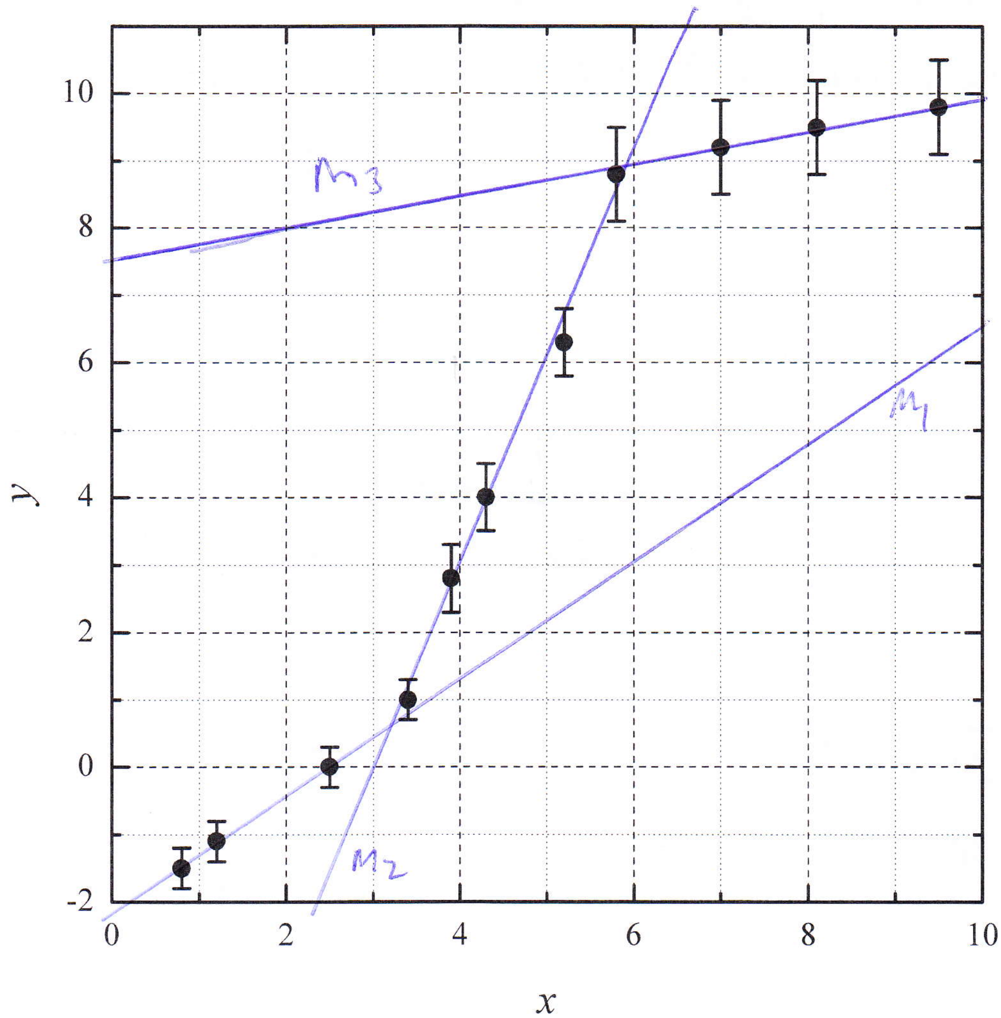
$$\boxed{\sigma_W = \sqrt{(m\sigma_{k_B})^2 + (k_B\sigma_m)^2}}$$

0 pts

Sol'ns.

(10[pts]) **6.** An experimental physicist has collected data $y \pm \sigma_y$ versus $x \pm \sigma_x$ as shown in the table below and as plotted in the figure below. In the plot only the y-error bars are shown.

x	σ_x	y	σ_y
0.8	0.4	-1.5	0.3
1.2	0.4	-1.1	0.3
2.5	0.4	0.0	0.3
3.4	0.4	1.0	0.3
3.9	0.4	2.8	0.5
4.3	0.4	4.0	0.5
5.2	0.4	6.3	0.5
5.8	0.4	8.8	0.7
7.0	0.4	9.2	0.7
8.1	0.4	9.5	0.7
9.5	0.4	9.8	0.7



10 pts

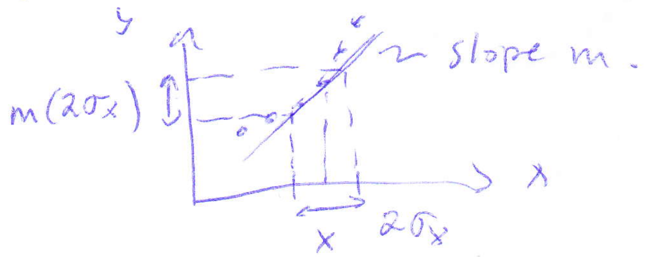
Sol'ns

The physicist has a theoretical model that she wants to fit to the data and she wants the fit to be a *weighted* fit. She thinks that for this dataset the errors in the x measurements are not negligible and wants to include their contributions while using the standard weighted fit procedure.

(a) Describe in a few short sentences the procedure that can be used to include the contributions of the σ_x uncertainties when doing the weighted fit. (**3 marks**)

(b) When completing a weighted fit, the weighting used for data point $(x_i \pm \sigma_{x,i}, y_i \pm \sigma_{y,i})$ is $1/\sigma^2_{\mathrm{net},i}$. Determine σ_{net} for the three points $(1.2 \pm 0.4, -1.1 \pm 0.3)$, $(4.3 \pm 0.4, 4.0 \pm 0.5)$, and $(8.1 \pm 0.4, 9.5 \pm 0.7)$. (**7 marks**)

(a) need to estimate contribution of σ_x to uncertainty along ~~the~~ y-dir'n. This is done by multiplying σ_x by slope of tangent line at value of x that is of interest



Then net uncertainty in y-dir'n is

$$\sigma_{net} = \sqrt{\sigma_y^2 + (m\sigma_x)^2}$$

(b) $m_1 = \dfrac{3-(-0.5)}{6-2} = \dfrac{3.5}{4} = 0.875$ $\therefore \sigma_{net} = \sqrt{(0.3)^2 + (0.4)^2(0.875)^2}$

$= 0.46 \approx \boxed{0.5}$

$m_2 = \dfrac{11-0}{6.5-3} = \dfrac{11}{3.5} \simeq 3.14$ $\therefore \sigma_{net} = \sqrt{(0.5)^2 + (0.4 \cdot 3.14)^2} = \boxed{1.35}$

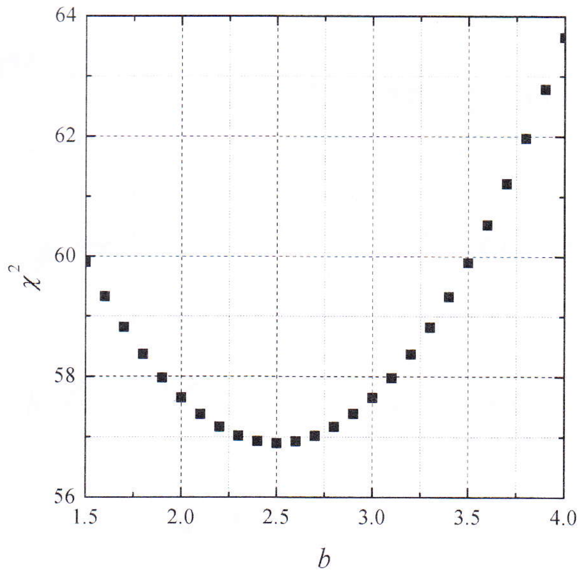
$\approx \sigma_x m_2$

$m_3 = \dfrac{10-8}{10-2} = \dfrac{2}{8} = 0.25$ $\therefore \sigma_{net} = \sqrt{(0.7)^2 + (0.25 \cdot 0.4)^2}$

$\boxed{= 0.71} \approx \sigma_y$

0 pts

(10^{pts}) **7.** Suppose that a quantity $y(x)$ as been measured as a function of x and that y also depends on a set of parameters a, b, and c such that $y = y(x; a, b, c)$. The exact form of y does not matter for this problem, but as an example, the function could be $y = a\sin(x/b) + c$. The data are next fit to the model and the best-fit parameters of a, b, and c are determined. The experimenter next tries to estimate the uncertainty in each of the parameters. In the figure below, parameters a and c are fixed at their best-fit values and the calculated χ^2 is shown as a function of b.



(a) Write down the general expression for χ^2. (**2 marks**)

(b) When collecting the y versus x dataset N measurements were collected and reasonable estimates of σ_y were made. Based on the plot above, estimate the value of N. Explain your reasoning. (**3 marks**)

(c) Based on the plot above, estimate the best-fit value of b and its uncertainty σ_b. Explain your reasoning. (**5 marks**)

(a) $$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - y(x_i)}{\sigma_i}\right)^2$$

(b) on avg. expect that $y_i - y(x_i) \approx \sigma_i$ $\therefore \left(\frac{y_i - y(x_i)}{\sigma_i}\right)^2 \approx 1$

$\therefore$ expect $\chi^2 \approx \sum_{i=1}^{N} 1 = N$

$\therefore$ min value of χ^2 gives approximately the no. of measurements in the data set.

$\therefore$ Expect $N \approx 57$. (Really $\chi^2 \approx N - \nu$ where ν is no. of degrees of freedom

10 pts

Sol'ns

For a fit to 3-parameters, $\nu = 3$

$\therefore \chi^2 = 57 = N - 3 \quad \therefore N \simeq 60$).

(c) best-fit value of b is the ~~one~~ value that minimizes χ^2. $\therefore b \simeq 2.5$

Near minimum ~~x~~ χ^2 varies quadratically ~~abo~~ w.r.t b

$\chi^2 \simeq \chi_0^2 + B(b-b^*)^2$. In class showed that $\sigma_b^2 = \frac{1}{B}$

$$\therefore \chi^2 = \chi_0^2 + \frac{(b-b^*)^2}{\sigma_b^2}.$$

$\therefore$ need to find B to estimate σ_b.

$B = \frac{\chi^2 - \chi_0^2}{(b-b^*)^2}$ take $\chi_0^2 = 56.9$ $b^* = 2.5$
$\chi^2 = 57.9$ $b = 1.9$ $\therefore B \simeq 2.78$

$$\therefore \sigma_b = \frac{1}{\sqrt{B}} = 0.6$$

$$\boxed{\therefore b = 2.5 \pm 0.6}$$

Alternatively, if $\chi^2 - \chi_0^2 = 1 = \frac{(b-b^*)^2}{\sigma_b^2}$

then $\sigma_b = (b - b^*)$

i.e. find b that increases χ^2 by 1 above ~~its~~ it's minimum value.

0 pts

$\cancel{x_0}$ $x^2 \to x_0^2 + 1$ when ~~b*~~ $b = 3.1$

$\therefore \sigma_b \approx |\cancel{b^*} b - b^*|$

$= |3.1 - 2.5| = 0.6$

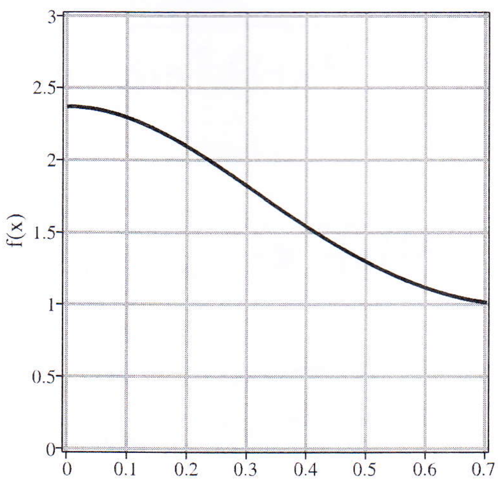
$\therefore \sigma_b = 0.6$ as before.

Sol'ns

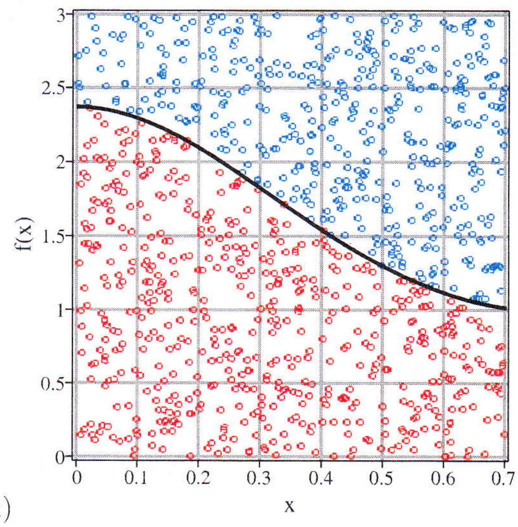
(10^{pts}) **8.** In this problem you will attempt to estimate the value of the following definite integral:

$$I = \int_0^{0.7} \frac{dx}{\sin^5(1+x^2)}.$$

This integral is not easily evaluated analytically, so the Monte Carlo Hit & Miss method will be used to numerically estimate I.



(i)



(ii)

(a) Briefly describe how the Monte Carlo Hit & Miss method is used to estimate the values of definite integrals. (~~2~~ 3 **marks**)

(b) In figure (i) above the function $f(x) = \sin^{-5}(1+x^2)$ is plotted over the interval $0 \le x \le 0.7$. Figure (ii) shows an implementation of the hit & miss method. $N = 1000$ trials were attempted and they are all shown in the figure. The number of hits (red ~~points~~ squares) recorded was $Z_N = 558$. What is the numerically determined value of I from this simulation? (**4 marks**)

(c) Estimate the uncertainty in the determination of I. (~~**4 marks**~~ 3 marks)

(a) Randomly select points inside a square area. The x-range of the square should correspond to the range of limits of integration. The y-range of square should completely contain the fcn. Prob. that randomly selected pts falls below the curve of fcn is $\frac{A_f}{A_{sq}}$ where A_f is area below fcn & A_{sq} is area of square. This prob. is numerically estimated by randomly generating N pts & counting the hits Z_N.

$$p = \frac{Z_N}{N} = \frac{A_f}{A_{sq}} \quad \therefore A_f = \boxed{\int_{x_1}^{x_2} f(x)\,dx = A_{sq}\frac{Z_N}{N}}$$

10 pts

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Sol'ns.

(b) $A_{sq} = (0.7)(3) = 2.1$

$$\therefore I = \int_0^{0.7} \frac{dx}{\sin^5(1+x^2)} = A_{sq}\frac{Z_n}{N} = 2.1\,\frac{558}{1000} = 1.1718$$

(c) error in Z_n determined from binomial dist'n

$$\sigma_{Z_n}^2 = np(1-p) \quad \text{where } n = 1000$$

$$p \approx \frac{Z_n}{n} = 0.558$$

$$\therefore \sigma_{Z_n} \simeq 15.7 = 16$$

$\sigma_I = \frac{A_{sq}}{N}\sigma_{Z_n}$ by prop. of errors.

$$\therefore \boxed{\sigma_I = 0.033}$$

$$\therefore I = 1.17 \pm 0.03$$